Chapter 5.4

Sampling Distributions & Central Limit Theorem

RANDOM NUMBER TABLE

19267	95457	53497
71549	44843	26104
27386	50004	05358
76612	39789	13537
90379	51392	55887
94456	48396	73780
94730	95761	75023
93621	66330	33393
34449	63513	83834
73920	56297	72678

If we averaged all responses in class $\mu_{\bar{x}}$ should come out close to μ

Mean: $\mu = 4.787$

Randomly choose a block of 5 numbers. Circle it.

Find the mean of your sample: $\bar{x} = \frac{0+5+3+5+8}{5} = 4.2$

A **sampling distribution** is the probability of a sample statistic that is formed when samples of size *n* are repeatedly taken from a population. In our case we would be looking at a "**sampling distribution of sample means.**"

The "mean of the means" (The average of our classroom results) would be written:

$$\mu_{\bar{x}}$$

The standard deviation of our sample means would be written:

$$\sigma_{\bar{x}}$$

PROBABILITY OF A MEAN RATHER THAN A SINGLE SCORE

Example 1: Children between the ages of 2 and 5 watch an average of 25 hours of TV per week with a standard deviation of 3 hours. If a sample of 20 children are selected, find the probability that the mean number of hours watched per week will be greater than 26.3?

μ σ n

Variables in this problem:

μ = mean = 25

n = sample size = 20

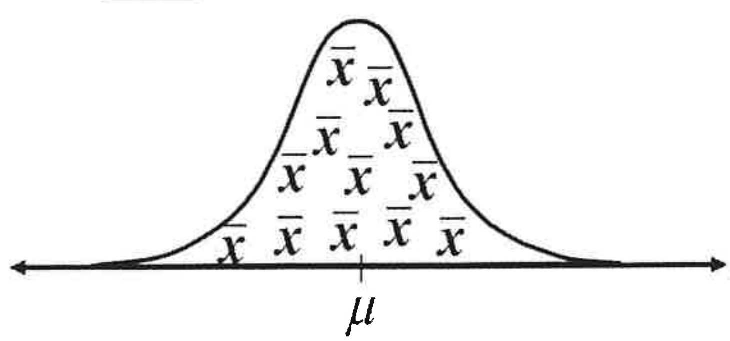
σ = standard deviation = 3

$\bar{x}$ = mean of sample = 26.3

CENTRAL LIMIT THEOREM

(Summary)

For ANY population of size "$n \geq 30$" or for populations known to be normally distributed, the sampling distribution can be approximated by the normal distribution where:



$$\mu_{\bar{x}} = \mu \quad and \quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$ ☆

Know what these mean

$\mu_{\bar{x}}$ = mean of sample means

$\sigma_{\bar{x}}$ = standard deviation of sample means

The Central Limit Theorem forms the foundation for the Inferential branch of statistics. This theorem describes the relationship between the sampling distribution of sample means and the population that the samples are taken from. The Central Limit Theorem is an important tool that provides the information you need to use sample statistics to make inferences about a population mean.

The Central Limit Theorem

1. If random samples of size n, where $n \geq 30$, are drawn from any population with a mean μ and a standard deviation σ, then the sampling distribution of sample means approximates a normal distribution. The greater the sample size, the better the approximation. (See figures for "Any Population Distribution" below.)
2. If random samples of size n are drawn from a population that is normally distributed, then the sampling distribution of sample means is normally distributed for *any* sample size n. (See figures for "Normal Population Distribution" below.)

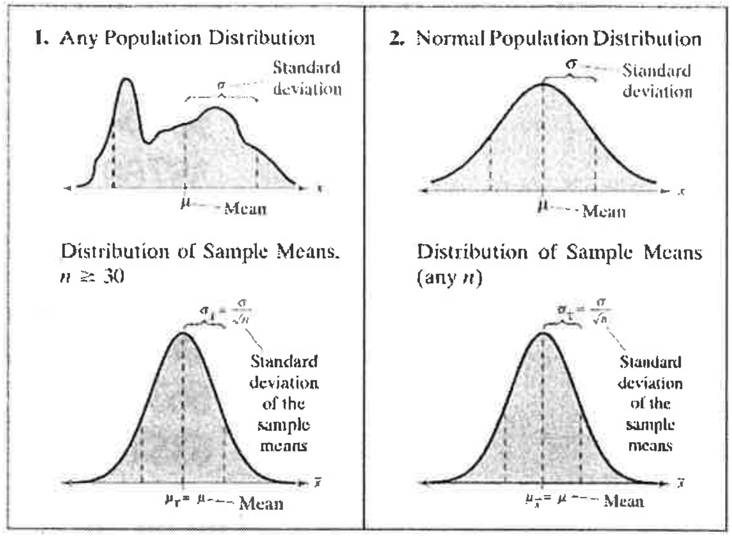
In either case, the sampling distribution of sample means has a mean equal to the population mean.

$\mu_{\bar{x}} = \mu$ Mean of the sample means

The sampling distribution of sample means has a variance equal to $1/n$ times the variance of the population and a standard deviation equal to the population standard deviation divided by the square root of n.

$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}$ Variance of the sample means

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$ Standard deviation of the sample means



Given the mean and the standard deviation of the population, find the mean and standard deviation of the sampling distribution of sample means with sample size *n*.

Example 2: a. $\mu = 28,\ \sigma = 3,\ n = 40$

$\mu_{\bar{x}} = 28$

$\sigma_{\bar{x}} = \frac{3}{\sqrt{40}} = .47$

b. $\mu = 1200,\ \sigma = 125,\ n = 1000$

$\mu_{\bar{x}} = 1200$

$\sigma_{\bar{x}} = \frac{125}{\sqrt{1000}} = 3.95$

This means we can still use our table if we adjust our calculation for Z!

$$\boxed{z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$$

← use if from a sample

Example 3: Calculator Practice
Matching: Use the given information to compute a Z-score.

$$\boxed{z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$$

a) $\bar{x} = 29,\ \mu = 30,\ \sigma = 5,\ n = 35$

$z = \frac{(29-30)}{(5/\sqrt{35})} \approx -1.18$ B

b) $\bar{x} = 15,\ \mu = 12.5,\ \sigma = 3.8,\ n = 10$

$z = \frac{(15-12.5)}{(3.8/\sqrt{10})} \approx 2.08$ A

c) $\bar{x} = 110,\ \mu = 95,\ \sigma = 34,\ n = 20$

$z = \frac{(110-95)}{(34/\sqrt{20})} \approx 1.97$ C

A. 2.08
B. -1.18
C. 1.97

Finding Probabilities

Example 4: The population mean and standard deviation are given. Find the indicated probability and determine whether the given sample mean would be considered unusual.

a. For a random sample of $n = 81$, find the probability of a sample mean being less than 45.3 when $\mu = 45$ and $\sigma = 2.3$

$$z = \frac{(45.3 - 45)}{\left(\frac{2.3}{\sqrt{81}}\right)} \approx 1.17$$

$P(\bar{x} < 45.3) = P(z < 1.17) = .8790$

Not unusual

b. For a random sample of $n = 60$, find the probability of a sample mean being greater than 293 when $\mu = 300$ and $\sigma = 25.5$

$$z = \frac{293 - 300}{\frac{25.5}{\sqrt{60}}} \approx -2.13$$

$P(z > -2.13) \approx 1 - .0166 = .9834$

not unusual.

Using and Interpreting the Central Limit Theorem

Example 5: Find the mean and standard deviation of the indicated sampling distribution of sample means. Then sketch a graph of the sampling distribution.

The amounts of cold water for patient consumption in hospitals in Spain are normally distributed, with a mean of 196 cubic meters per bed and a standard deviation of 70 cubic meters per bed. Random samples of size 15 are drawn from this population, and the mean of each sample is determined.

$\mu = 196$ $\sigma = 70$ $n = 15$

$\mu_{\bar{x}} = 196$

$$\sigma_{\bar{x}} = \frac{70}{\sqrt{15}} = 18.07$$

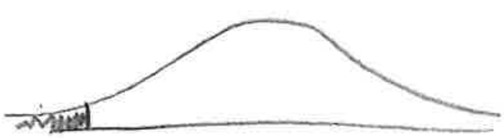
Finding Probabilities for Sampling Distributions

Example 6: A certain kind of bacteria exists in water. Let x be the bacteria count per milliliter of water. The health department has found that if the water is not contaminated, then x has a distribution that is more or less mound-shaped and symmetrical. The mean of x is $\mu = 3000$ and the standard deviation is $\sigma = 200$. The city health inspector takes 64 random samples of water from the city public water system each day. Assuming the water is not contaminated, what is the probability that $\bar{x}$ is less than 2925?

$\mu = 3000$ $\sigma = 200$ $n = 64$ $\bar{x} < 2925$

$$z = \frac{2925 - 3000}{\frac{200}{\sqrt{64}}} = -3.00$$

$P(\bar{x} < 2925) = P(z < -3.00) = .0013$



Example 7: The average sales price of a single-family house in the United States is \$176,800. You randomly select 12 single-family houses. What is the probability that the mean sales price is more than \$160,000? Assume that the sales prices are normally distributed with a standard deviation of \$50,000.

$\mu = 176800$ $\quad \sigma = 50000$ $\quad \bar{x} = 160000$ $\quad z = \dfrac{160000 - 176800}{\frac{50000}{\sqrt{12}}} = -1.16$

$n = 12$

$P(\bar{x} > 160000) = P(z > -1.16) = 1 - .1230 = .8770$

Example 8: Is it Unusual?

The weights of ice cream cartons produced by a manufacturer are normally distributed with a mean weight of 10 ounces and a standard deviation of 0.5 ounces.

$\mu = 10 \quad \sigma = .5$

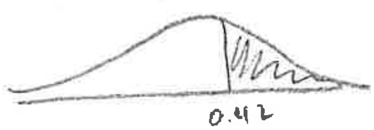
a) What is the probability that a randomly selected carton has a weight greater than 10.21 ounces? Does this seem unusual?

This is just 1 - so like we did in earlier sections

$P(x > 10.21) = P(z > 0.42) \approx 1 - .6218 = .3782$

Not unusual

$z = \dfrac{10.21 - 10}{.5} = .42$



b) You select 25 cartons. What is the probability that their mean weight is greater than 10.21 ounces? Does this seem unusual? $n = 25$

$P(\bar{x} > 10.21) = P(z > 2.10) = 1 - .9821$

$= .0179$

unusual

$z = \dfrac{10.21 - 10}{\frac{.5}{\sqrt{25}}} = 2.10$

Example 9: Is it Unusual?

A machine is set to fill paint cans with a mean of 128 ounces and a standard deviation of 0.2 ounce. A random sample of 40 cans has a mean of 127.9 ounces. Does the machine need to be reset?

$\mu = 128 \quad \sigma = .2 \quad n = 40 \quad \bar{x} = 127.9$

$\dfrac{127.9 - 128}{\frac{.2}{\sqrt{40}}} = -3.16$

$P(z < -3.16) = .0008$

Yes, very unusual, so the machine should be reset.